# CERENKOV RADIATION PRODUCED BY 100 MeV ELECTRONS

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# NAVAL POSTGRADUATE SCHOOL
## Monterey, California

# THESIS

CERENKOV RADIATION PRODUCED BY
100 MeV ELECTRONS

by

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June 1981

Thesis Advisor: Fred R. Buskirk

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T200059

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SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

| REPORT DOCUMENTATION PAGE | | READ INSTRUCTIONS BEFORE COMPLETING FORM |
|---|---|---|
| 1. REPORT NUMBER | 2. GOVT ACCESSION NO. | 3. RECIPIENT'S CATALOG NUMBER |
| 4. TITLE (and Subtitle)<br>Cerenkov Radiation Produced by 100 MeV Electrons | | 5. TYPE OF REPORT & PERIOD COVERED<br>Master's Thesis; June 1981<br>6. PERFORMING ORG. REPORT NUMBER |
| 7. AUTHOR(s)<br>David Earl McLaughlin | | 8. CONTRACT OR GRANT NUMBER(s) |
| 9. PERFORMING ORGANIZATION NAME AND ADDRESS<br>Naval Postgraduate School<br>Monterey, California 93940 | | 10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS |
| 11. CONTROLLING OFFICE NAME AND ADDRESS<br>Naval Postgraduate School<br>Monterey, California 93940 | | 12. REPORT DATE<br>June 1981<br>13. NUMBER OF PAGES<br>48 pages |
| 14. MONITORING AGENCY NAME & ADDRESS(if different from Controlling Office) | | 15. SECURITY CLASS. (of this report)<br>Unclassified<br>15a. DECLASSIFICATION/DOWNGRADING SCHEDULE |

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Stimulated Cerenkov radiation, superluminal, LINAC beam monitor.

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

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S/N 0102-014-6601 |
(Page 1)



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of stimulated Cerenkov radiation in a dielectric slab. The first experimental results using extremely relativistic electrons are reported, and the problems encountered are outlined with some suggestions for improvements. The results of this validation show that the observed frequency differs from the predicted frequency by less than 1.5%. Incidental to the conduct of this experiment, ordinary Cerenkov radiation in the usual cone was observed in air at microwave frequencies. A possible application of the stimulated Cerenkov process as an electron beam monitor is briefly discussed.

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# Cerenkov Radiation Produced by 100 MeV Electrons

by

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Lieutenant Commander, United States Navy
B.S.M.E., Michigan State University, 1969

Submitted in partial fulfillment of the
requirements for the degree of

MASTER OF SCIENCE IN PHYSICS

from the

NAVAL POSTGRADUATE SCHOOL
June 1981

ABSTRACT

It is proposed that electromagnetic radiation of a specified frequency can be produced as a result of stimulated Cerenkov radiation in a dielectric resonator excited by a superluminal electron beam. The frequency generated is a function of three physical parameters. They are the electron energy, the thickness of the dielectric resonator and its index of refraction. This work provides a theoretical derivation for predicting the frequency of stimulated Cerenkov radiation in a dielectric slab. The first experimental results using extremely relativistic electrons are reported, and the problems encountered are outlined with some suggestions for improvements. The results of this validation show that the observed frequency differs from the predicted frequency by less than 1.5%. Incidental to the conduct of this experiment, ordinary Cerenkov radiation in the usual cone was observed in air at microwave frequencies. A possible application of the stimulated Cerenkov process as an electron beam monitor is briefly discussed.

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ACKNOWLEDGEMENTS

The author wishes to express his gratitude to his thesis advisor, Professor Fred R. Buskirk, for his instruction, guidance and assistance in all phases of this project.

Thanks are also extended to Professor J. Knorr of the Naval Postgraduate School's Electrical Engineering Department for providing equipment and for his many helpful suggestions in the course of this work, and to Professor John E. Walsh of Dartmouth College for his encouragement.

The assistance of Mr. Don Snyder in the fabrication of the experimental setup and in the day-to-day operation and maintenance of the NPS LINAC is greatly appreciated.

Finally, the author wishes to thank his wife, Catherine, for her understanding, patience and support throughout this work and for her assistance in composition and proofreading.

# I. INTRODUCTION

Cerenkov radiation, the radiation which is generated by a charged particle moving at superluminal velocity in a medium has been considered as a possible radiation source for many years. Additionally, Cerenkov radiation has been used as a charged particle detector which will selectively detect only charged particles exceeding the speed of light in the medium through which they are traveling. This detection is dependent upon only the velocity of the particle and the index of refraction of the medium. The detector is a simple photocell and as such does not distinguish between frequencies within its band of operability. Recently, Professor John E. Walsh [Ref. 1] reported successful generation of Cerenkov radiation using relativistic electrons and a dielectric resonator in the form of a cylindrical anulus. The import of Walsh's work is that the frequency of the radiation generated in the dielectric is a function only of the energy of the electrons, the thickness of the dielectric, and the index of refraction of the dielectric. Since these quantities are material parameters, it would appear that radiation of any desired frequency could be obtained by selecting the proper set of parameters.

The intent of this experiment is to extend Walsh's work, using a different geometry for the dielectric resonator and

much higher electron energies. Professor Walsh conducted his experiments at 300 KeV in contrast to the present experiment, which uses electron energies in the 10-100 MeV range, an increase of 1.5 to 2 orders of magnitude. In this experiment, radiation in the X band was selected for ease of measurement and analysis, with a view to extending the work into the IR range.

## II. THEORY

Consider a dielectric slab with a coordinate system as shown in Figure 1. The dielectric has thickness h in the z direction and is unbounded in the x and y directions. There is a dielectric-conductor interface at z=0 which lies in the x-y plane. The dielectric slab has permittivity $\varepsilon$ and permeability $\mu_0$ while these quantities are $\varepsilon_0$ and $\mu_0$, respectively, in vacuum.

Starting with Maxwell's equations and following the usual procedure, it can be shown that the longitudinal components, $E_x$ or $H_x$, satisfy a wave equation and the modes can be divided into TE or TM modes. The general expressions for the transverse components of the wave equation solution are:

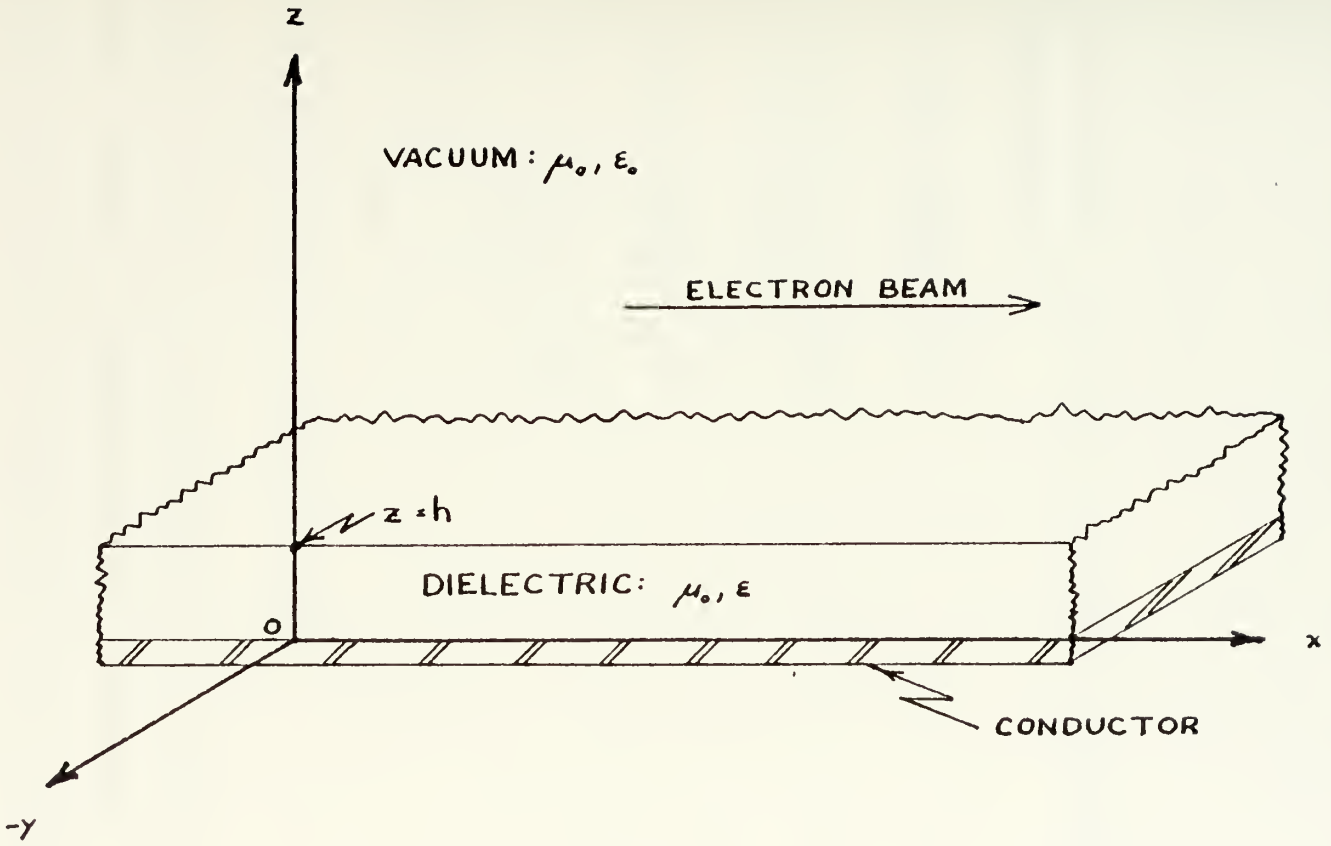
$$H_y = \frac{i\omega\varepsilon}{\omega^2\mu\varepsilon-k^2}\frac{\partial E_x}{\partial z} - \frac{ik}{\omega^2\mu\varepsilon-k^2}\frac{\partial H_x}{\partial y} \tag{1}$$

$$F_z = \frac{-ik}{\omega^2\mu\varepsilon-k^2}\frac{\partial E_x}{\partial z} + \frac{i\omega\mu}{\omega^2\mu\varepsilon-k^2}\frac{\partial H_x}{\partial y} \tag{2}$$

$$E_y = \frac{-i\omega\mu}{\omega^2\mu\varepsilon-k^2}\frac{\partial H_x}{\partial z} - \frac{ik}{\omega^2\mu\varepsilon-k^2}\frac{\partial E_x}{\partial y} \tag{3}$$

$$H_z = \frac{-ik}{\omega^2\mu\varepsilon-k^2}\frac{\partial H_x}{\partial z} - \frac{i\omega\varepsilon}{\omega^2\mu\varepsilon-k^2}\frac{\partial E_x}{\partial y} \tag{4}$$

The intent of this experiment is to take energy from a beam of electrons in the vacuum just above the dielectric slab




COORDINATE SYSTEM

FIGURE 1.

and give this energy to an EM wave in the slab and in the vacuum above the slab. This energy transfer requires a component of $\vec{E}$ in the x-direction. Therefore, the wave in the dielectric should be TM mode. Thus, for TM modes where $H_x=0$ and $E_x$ exist, the wave equation for $E_x$ is:

$$(\nabla_\perp^2 + \{\omega^2\mu\varepsilon-k^2\})E_x = 0 \tag{5}$$

When the condition that $H_x=0$ is applied to Equations 1, 2, 3 and 4, the result is

$$H_y = \frac{i\omega\varepsilon}{\omega^2\mu\varepsilon-k^2}\frac{\partial E_x}{\partial z} \tag{6}$$

$$E_z = \frac{-ik}{\omega^2\mu\varepsilon-k^2}\frac{\partial E_x}{\partial z} \tag{7}$$

$$E_y = \frac{-ik}{\omega^2\mu\varepsilon-k^2}\frac{\partial E_x}{\partial y} \tag{8}$$

$$H_z = \frac{-i\omega\varepsilon}{\omega^2\mu\varepsilon-k^2}\frac{\partial E_x}{\partial y} \tag{9}$$

For simplicity, let

$$\omega^2\mu_0\varepsilon_0-k^2 = -a^2 \tag{10}$$

for waves propagating in vacuum and let

$$\omega^2\mu_0\varepsilon-k^2 = b^2 \tag{11}$$

for waves propagating in the dielectric. Assuming no y dependence, which is appropriate for a slab which extends to infinity in the + and - y directions, Eq. 5 reduces to

$$\frac{\partial^2 E_x}{\partial z^2} + (\omega^2\mu\varepsilon-k^2)E_x = 0 \tag{12}$$

Thus, in vacuum, substituting Eq. 10 into Eq. 12 yields

$$\frac{\partial^2 E_x}{\partial z^2} - a^2 E_x = 0 \tag{13}$$

which, when the restriction that $E_x$ goes to zero as z goes infinity is applied, has as its solution

$$E_x = Ae^{-az} \tag{14}$$

Similarly, substituting Eq. 11 into Eq. 12 for the dielectric case yields

$$\frac{\partial^2 E_x}{\partial z^2} + b^2 E_x = 0 \tag{15}$$

When the boundary condition of a conductor at the origin is imposed, the solution to Eq. 15 is

$$E_x = B \sin (bz) \tag{16}$$

At the vacuum-dielectric interface, the usual boundary conditions apply. That is, the tangential components of $\vec{E}$ and $\vec{H}$ must be continuous and the normal components of $\vec{D}$ and $\vec{B}$ must also be continuous. For this problem, with the given coordinate system, the tangential components of $\vec{E}$ are $E_x$ and $E_y$, the tangential components of $\vec{H}$ are $H_x$ and $H_y$. The normal component of $\vec{D}=\varepsilon\vec{E}$ is $\varepsilon E_z$ and the normal component of $\vec{B}=(1/\mu)\vec{H}$ is $\frac{1}{\mu}H_z$. $E_y$ and $H_y$ are zero when the assumption of no y dependence is applied to Eqs. 8 and 9 and for TM modes $H_x$ is zero by definition. Thus, the boundary conditions reduce to: $E_x$, $H_y$ and $\varepsilon E_z$ must all be continuous at the

vacuum-dielectric interface. Equating Eqs. 14 and 16 at z=h and applying the boundary condition of continuity to $E_x$ leads to

$$Ae^{-ah} = B \sin(bh) \tag{17}$$

Making the appropriate substitutions into either Eqs. 6 or 7 and applying the continuity requirement for $\varepsilon E_z$ at z=h results in the same equation.

$$\frac{i\omega\varepsilon_0(-aAe^{-ah})}{-a^2} = \frac{i\omega\varepsilon(bB\cos\{bh\})}{b^2} \tag{18}$$

This reduces to

$$\frac{\varepsilon_0 Ae^{-ah}}{a} = \frac{\varepsilon B\cos(bh)}{b} \tag{19}$$

Equations 17 and 19 comprise a coupled set of equations which must be simultaneously satisfied.

The following development indicates one method of solution which leads to an expression for the propagation constant, k, and the corresponding phase velocity for the propagating mode. Dividing Eq. 17 by Eq. 19 gives

$$\frac{a}{\varepsilon_0} = \frac{b}{\varepsilon}\tan(bh) \tag{20}$$

$$\frac{\varepsilon}{\varepsilon_0}a = b\tan(bh) \tag{21}$$

$$\frac{\varepsilon}{\varepsilon_0}ah = bh\tan(bh) \tag{22}$$

Let y=ah and x=bh

$$y = \frac{\varepsilon_0}{\varepsilon}x\tan x \tag{23}$$

Applying the definition of the index of refraction (n)

$$y = \frac{x \tan x}{n^2} \tag{24}$$

Recalling the definitions of $a^2$ and $b^2$ and adding them

$$a^2 + b^2 = \omega^2\mu_0(\varepsilon-\varepsilon_0) \tag{25}$$

$$a^2h^2 + b^2h^2 = \omega^2\mu_0(\varepsilon-\varepsilon_0)h^2 \tag{26}$$

Substituting for $(ah)^2$ and $(bh)^2$

$$y^2 + x^2 = \omega^2\mu_0(\varepsilon-\varepsilon_0)h^2 \tag{27}$$

The right hand side of Eq. 26 is a constant, which we will call $R^2$

$$y^2 + x^2 = R^2 \tag{28}$$

The simultaneous solution of Eqs. 23 and 28 for a given frequency will result in the value of k which yields fields satisfying all boundary conditions. A representative solution of this system of equations with n=1.461 (polyethylene) is shown in Figure 2.

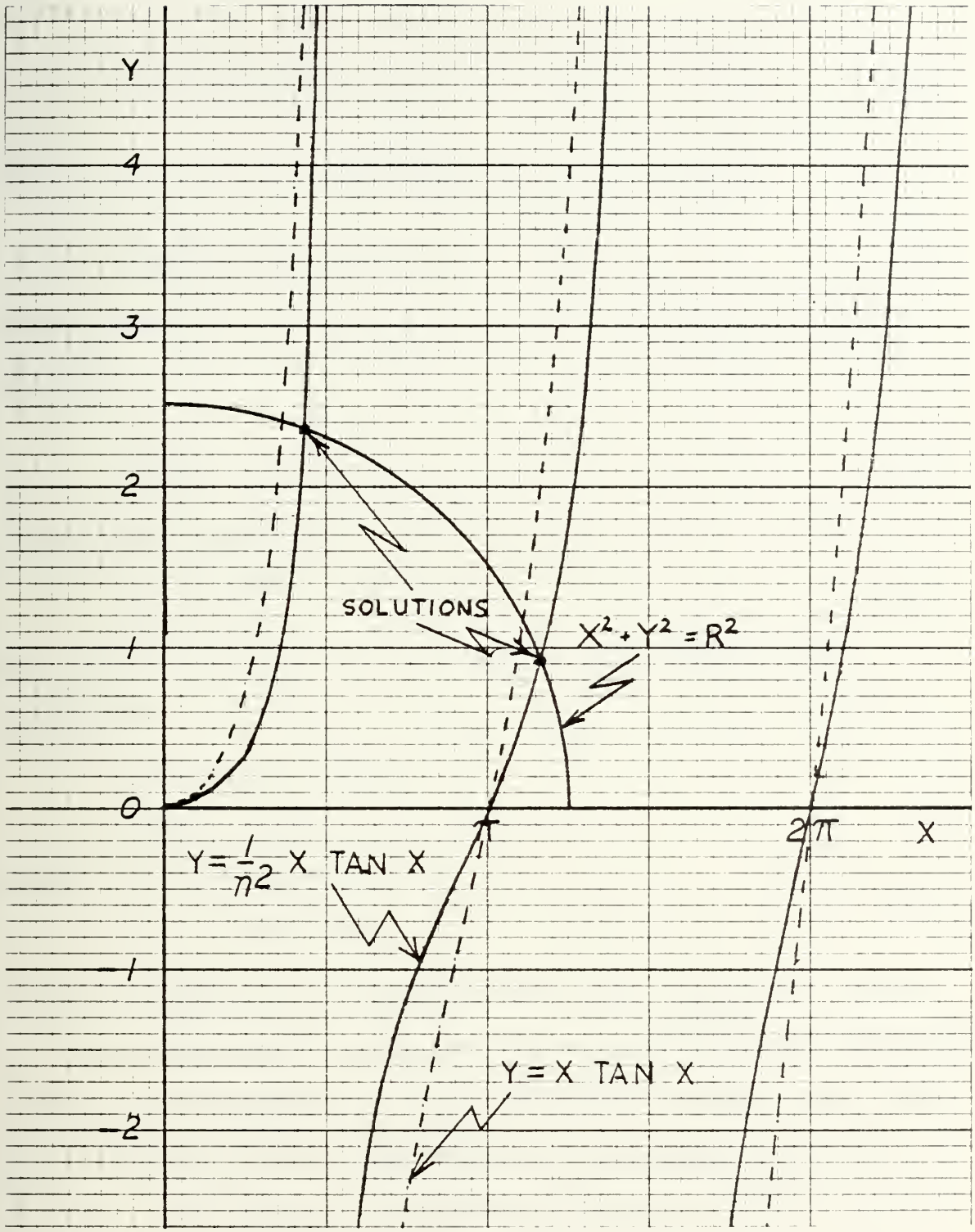
Although the TM modes are the desired modes, it is necessary to also consider the TE modes to see if there is any degeneracy. Equations 1, 2, 3 and 4 reduce to

$$H_y = \frac{-ik}{\omega^2\mu\varepsilon-k^2}\frac{\partial H_x}{\partial y} \tag{29}$$

$$E_z = \frac{i\omega\mu}{\omega^2\omega\varepsilon-k^2}\frac{\partial H_x}{\partial y} \tag{30}$$

$$E_y = \frac{-i\omega\mu}{\omega^2\mu\varepsilon-k^2}\frac{\partial H_x}{\partial z} \tag{31}$$

$$H_z = \frac{-ik}{\omega^2\mu\varepsilon-k^2}\frac{\partial H_x}{\partial z} \tag{32}$$




GRAPHICAL SOLUTION

FIGURE 2.

The TE mode wave equation for $H_x$ is

$$(\nabla_\perp^2 + \{\omega^2\mu\varepsilon - k^2\})H_x = 0 \tag{33}$$

Again, assuming no y dependence, Eq. 33 becomes

$$\frac{\partial^2 H_x}{\partial z^2} + (\omega^2\mu\varepsilon - k^2)H_x = 0 \tag{34}$$

Substituting Eq. 10 into Eq. 34 for waves propagating in vacuum

$$\frac{\partial^2 H_x}{\partial z^2} - a^2 H_x = 0 \tag{35}$$

which has a solution

$$H_x = Ce^{-az} \tag{36}$$

and substituting Eq. 11 into Eq. 34 for waves is the dielectric

$$\frac{\partial^2 H_x}{\partial z^2} + b^2 H_x = 0 \tag{37}$$

which has a solution

$$H_x = D \sin(bz) \tag{38}$$

when the appropriate boundary condition at z=0 is imposed.

The same requirements for the vacuum-dielectric interface apply. The assumption of no y dependence immediately makes Eqs. 29 and 30 zero. The remaining tangential component of $\vec{H}$ is $H_x$, the remaining tangential component of $\vec{E}$ is $E_y$ and for the normal components of $\vec{D}$ and $\vec{B}$, only $B_z = \frac{1}{\mu}H_z$ remains. These three components must be continuous at the interface

at z=h. Thus

$$Ce^{-ah} = D \sin (bh) \tag{39}$$

and

$$\frac{-i\omega\mu_0 \ (-aCe^{-ah})}{-a^2} = \frac{-i\omega\mu_0 \ (bD \cos \{bh\})}{b^2} \tag{40}$$

$$\frac{Ce^{-ah}}{a} = \frac{D \cos (bh)}{b} \tag{41}$$

Dividing Eq. 39 by Eq. 41,

$$a = b \tan (bh) \tag{42}$$

$$ah = bh \tan (bh) \tag{43}$$

Again. letting y=ah and x=bh,

$$y = x \tan x \tag{44}$$

Equations 27 and 28 are still valid and the simultaneous solution of Eqs. 28 and 44 will satisfy all boundary conditions for a given frequency.

The simularity between Eq. 44 and Eq. 23 indicates that both TE and TM modes will propagate at about the same frequency in the dielectric. However, only the TM mode is capable of gaining energy from the electrons. Accordingly, if stimulated Cerenkov radiation is observed, it will be the TM modes that are excited. If, however, an attempt is made to amplify waves fed into the dielectric from some outside source, some mechanism must be found to exclude the TE mode waves from being introduced into the dielectric.

A clearer picture of the relationships can be seen by use of geometry. First, define

$$k_f^2 = \omega^2 \mu_0 \varepsilon_0 \tag{45}$$

the magnitude of the free space k vector for a plane wave and

$$k_d^2 = \omega^2 \mu_0 \varepsilon \tag{46}$$

the magnitude of the k vector for a plane wave in the dielectric. Substituting these into Eqs. 10 and 11, the defining equations for $a^2$ and $b^2$ respectively, yields after multiplication by $h^2$,

$$k_f^2 h^2 - k^2 h^2 = -a^2 h^2 \tag{47}$$

$$k_d^2 h^2 - k^2 h^2 = b^2 h^2 \tag{48}$$

Subtracting Eq. 47 from Eq. 48

$$R^2 = h^2 k_d^2 - h^2 k_f^2 \tag{49}$$

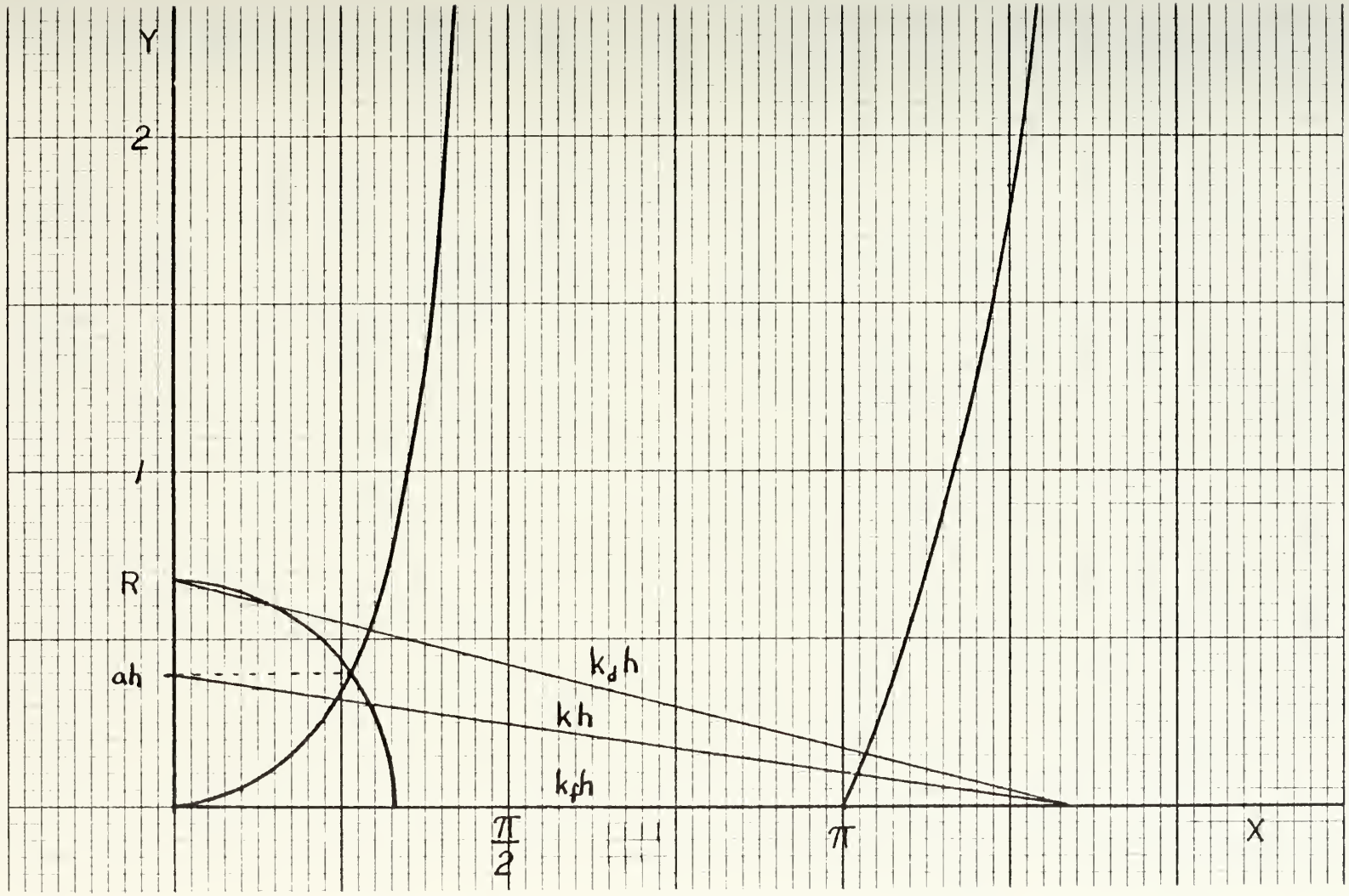
or

$$R^2 + h^2 k_f^2 = h^2 k_d^2 \tag{50}$$

This defines a right triangle as shown in Figure 2. Rearranging the terms in Eq. 47 defines another right triangle,

$$a^2 h^2 + k_f^2 h^2 = k^2 h^2 \tag{51}$$

also shown in Figure 3. Note that the side $k_f h$ is common to both triangles and that the magnitude of the unknown k, the wave number of the propagating mode in the dielectric that satisfies all the boundary conditions, is between $k_f$ and $k_d$. There is an important consequence of this development.




GEOMETRIC RELATIONSHIPS

FIGURE 3.

Since, for energy exchange, the phase velocity of the wave in the dielectric, as represented by the unknown, k, must be matched to the velocity of the electrons. Also the velocity of the electrons must be less than the speed of light in the medium above the dielectric. Thus the region above the dielectric must be in vacuum because 100 MeV electrons have a velocity slightly faster than the velocity of a plane wave in air under normal conditions.

At this point it is possible to proceed to a numerical solution. First, by phase matching the velocity of the electrons to the velocity of the stimulated wave in the dielectric

$$v_{el} = v_{mode} \tag{52}$$

By definition

$$\beta = \frac{v}{c} \tag{53}$$

and after applying Eq. 52

$$\beta c = v_{mode} \tag{54}$$

Again, by definition

$$k_{mode} = \frac{\omega}{v_{mode}} \tag{55}$$

$$k_{mode} = \frac{\omega}{\beta c} \tag{56}$$

and

$$\gamma^2 = \frac{1}{1-\beta^2} \tag{57}$$

$$\beta^2 = \left(1 - \frac{1}{\gamma^2}\right) \tag{58}$$

Substituting Eq. 58 into Eq. 56

$$k_{mode} = \frac{\omega}{(1 - \frac{1}{\gamma^2})^{\frac{1}{2}} c} \tag{59}$$

Thus, $k_{mode}$ is determined for a given $\gamma$ and $\omega$. Once $\omega$ is specified, Eqs. 45 and 46 specify $k_f$ and $k_d$. Rearranging Eq. 10 and substituting Eq. 45 yields

$$a^2 = k_{mode}^2 - k_f^2 \tag{60}$$

$$a = (k_{mode}^2 - k_f^2)^{\frac{1}{2}} \tag{61}$$

Similarly

$$b^2 = (k_d^2 - k_{mode}^2) \tag{62}$$

$$b = (k_d^2 - k_{mode}^2)^{\frac{1}{2}} \tag{63}$$

Solving Eq. 21 for h

$$\frac{\varepsilon a}{\varepsilon_0} = b \tan (bh) \tag{21}$$

$$\frac{\varepsilon a}{\varepsilon_0 b} = \tan (bh) \tag{64}$$

$$bh = \tan^{-1} (n^2 \frac{a}{b}) \tag{65}$$

$$h = \frac{1}{b} \tan^{-1} (n^2 \frac{a}{b}) \tag{66}$$

Thus, given a desired frequency, $\omega$, the index of refraction of the dielectric, n, and the energy of the electrons, $\gamma$, the thickness of the dielectric can be determined.

The substitution of Eq. 59 into Eqs. 10 and 11 allows the factoring out of $\omega^2$ and Eq. 66 becomes

$$h = \frac{1}{\omega\left(\mu_0\varepsilon - \frac{1}{(1-\frac{1}{\gamma^2})c^2}\right)^{\frac{1}{2}}} \tan^{-1}\left\{ n^2 \frac{\left(\frac{1}{(1-\frac{1}{\gamma^2})c^2} - \mu_0\varepsilon\right)^{\frac{1}{2}}}{\left(\mu_0\varepsilon - \frac{1}{(1-\frac{1}{\gamma^2})c^2}\right)^{\frac{1}{2}}} \right\} \quad (67)$$

which can be solved for $\omega$ given h, n, and $\gamma$.

$$\omega = \frac{1}{h\left(\mu_0\varepsilon - \frac{1}{(1-\frac{1}{\gamma^2})c^2}\right)^{\frac{1}{2}}} \tan^{-1}\left\{ n^2 \frac{\left(\frac{1}{(1-\frac{1}{\gamma^2})c^2} - \mu_0\varepsilon\right)^{\frac{1}{2}}}{\left(\mu_0\varepsilon - \frac{1}{(1-\frac{1}{\gamma^2})c^2}\right)^{\frac{1}{2}}} \right\} \quad (68)$$

The significance of Eq. 68 is that the physical parameters of h, n, and $\gamma$ specify $\omega$. With the proper selection of these parameters, EM radiation of any frequency can be generated.

Examination of Figure 2 readily shows that if the dimensionless parameter bh exceeds $\pi$, more than one solution is possible. However, these modes will be at different frequencies and the graphical solution indicates that, if $x<3\pi$, the single desired frequency can be extracted by use of a band-pass or high pass filter.

## III. EXPERIMENTAL EQUIPMENT AND PROCEDURE

It was decided to conduct this experiment in the X-band (8-12 GHz) after consulting with Professor Knorr and investigating the availability of the necessary equipment and measuring devices needed to support the experiment. The dielectric chosen was polyethelyne which has an index of refraction of 1.461 [Ref. 2]. The Naval Postgraduate School's Linear Accelerator (LINAC) is capable of producing relativistic electrons with energies up to 120 MeV. The operating characteristics of the NPS LINAC can be found in Appendix A.

A TI-59 programmable calculator program was developed to solve Eqs. 67 and 68. An explanation of this program and a listing of it can be found in Appendix B. Table I shows the input parameter values and the program's solution of Eq. 66 for the 0, 1 and 2 modes.

TABLE I

THICKNESSES OF POLYETHELYNE FOR MODES 0,1,2

| m | E [MeV] | n | f [FHz] | h[mm][1] |
|---|---|---|---|---|
| 0 | 50 | 1.461 | 10 | .095 |
| 1 | 50 | 1.461 | 10 | 14.07 |
| 2 | 50 | 1.461 | 10 | 28.15 |

[1]Program output value

Based on these results, a thickness of 12.7mm (0.5 in) was selected. The values of E=50 MeV, n=1.461, h=12.7mm for a Mode 1 solution predict a frequency of 11.08 GHz when the TI-59 program solves Eq. 68.

The beam of the LINAC is approximately 1 cm in diameter. The width of the dielectric was made to be 5.08 cm to approximate the infinite slab in the y direction. The end of the dielectric away from the LINAC exit window was tapered to fit into an X-band hollow waveguide and to facilitate the TM to TE transition required since a TE mode is the lowest mode that will propagate in a hallow waveguide. Figure 4 is a photograph of the dielectric resonator as it was originally configured.

The first attempt to observe the desired EM radiation met with only limited success. The dielectric slab was mounted about 1 cm below the centerline axis of the LINAC exit window in air. The metal waveguide was sloped slightly to allow clear passage of the electrons over the top of the flange connector to the detector. A diode detector was used with its output lead to an oscilliscope (CRO). With a beam energy of 53.85 MeV and an average beam current[2] of 8.0 nAmp, the CRO display showed a pulse of about 1 μsec with strength varying from 100-200 mV as the beam was tuned, focused and

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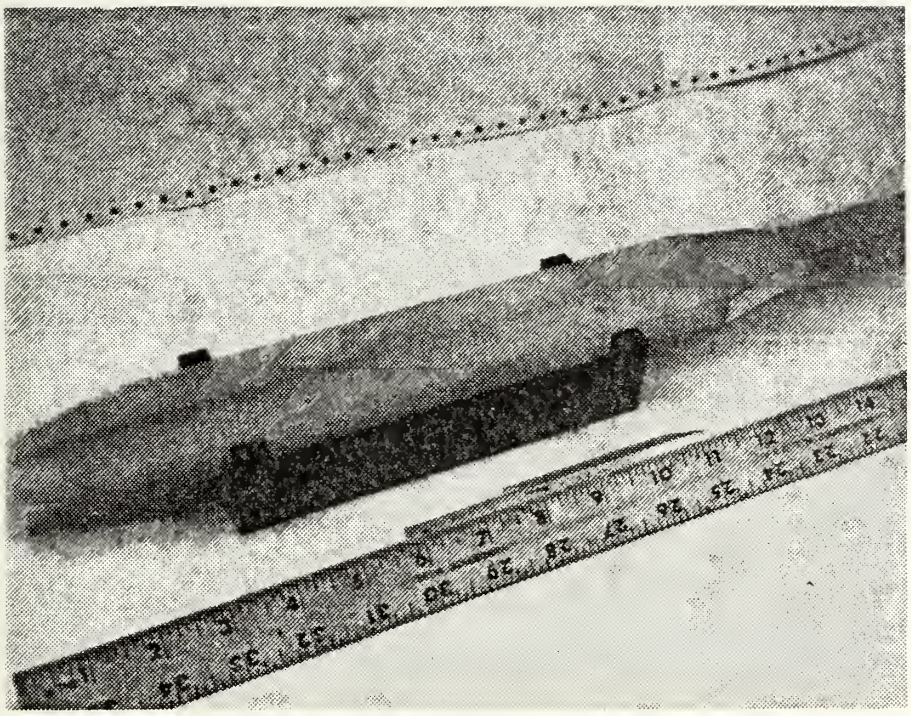
[2]All currents read on a secondary emission monitor with 6% efficiency, and the monitor current is reported in this work.



DIELECTRIC RESONATOR

FIGURE 4.

varied in distance from the slab. However, attempts to measure the frequency with a high Q calibrated cavity resonator were unsuccessful.

To determine if the observed pulse was caused by the dielectric, the slab was removed leaving the hollow waveguide in place. When the beam was turned on, a 0.5 microsecond pulse of slightly reduced magnitude was observed. At this point, it was realized that the observed pulse might be caused by Cerenkov radiation in air. Although commonly accepted as 1.0, the actual index of refraction of standard dry air is 1.003 [Ref. 3]. Using this value as $\beta^{-1}$ it can be easily shown that electrons with energies exceeding 20.856 MeV will exceed the speed of light in standard dry air. Thus, electrons above that energy would exceed the velocity of a plane wave in air, and could not be matched to the TM mode velocity.[3]

A simple experiment was designed to investigate this possibility. The equipment arrangement is shown in Figure 5. The reflector used was a thin sheet of aluminum which would allow the passage of the electrons and reflect any EM radiation. With the reflector in place, a pulse was observed from detector 2 whose magnitude was approximately 0.5 times the magnitude of the output from detector 1. When the reflector was removed, a pulse of magnitude 0.1 times the output of

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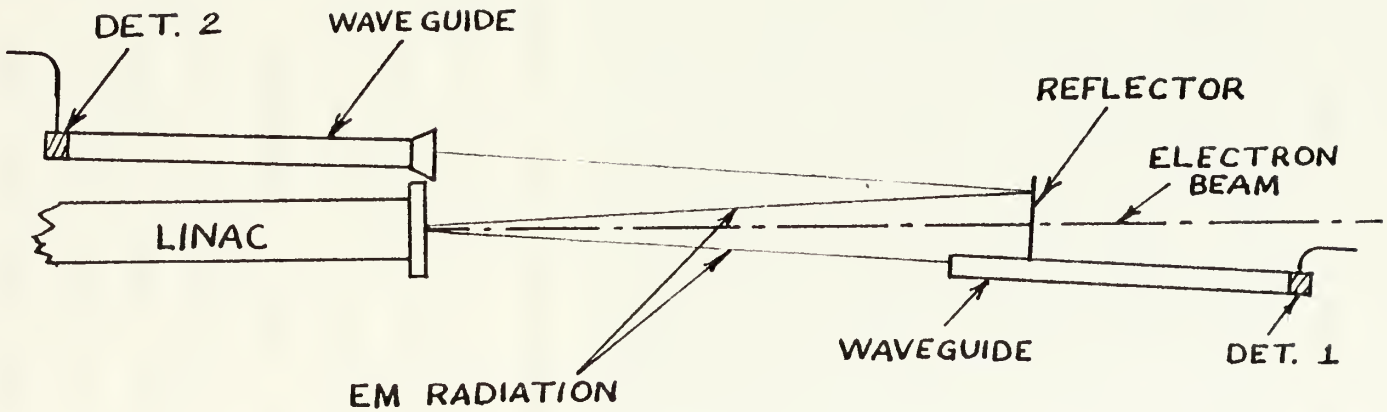
[3]See Section II.



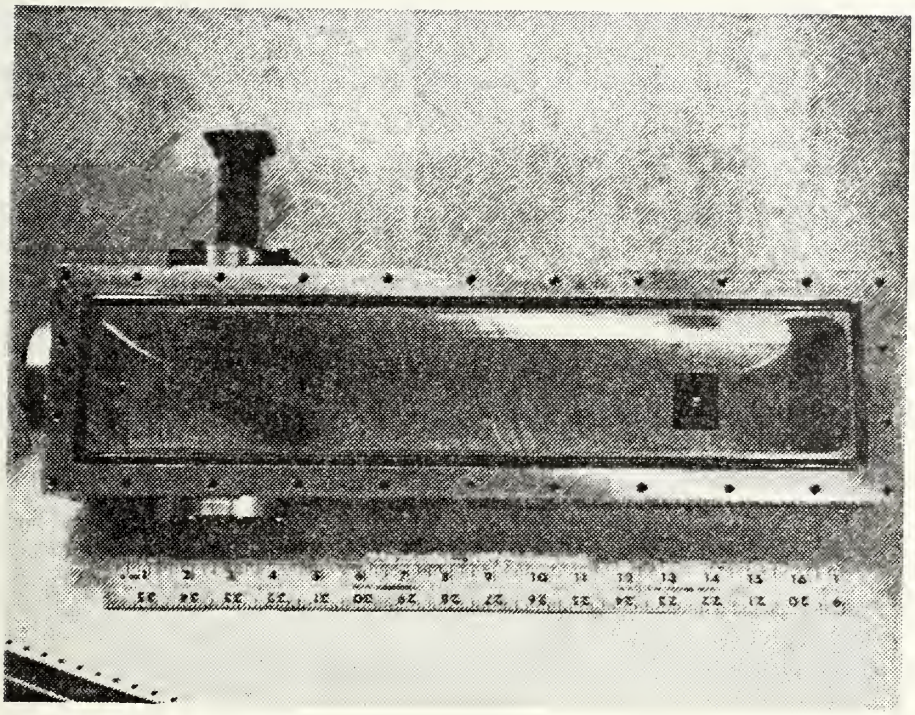

AIR CERENKOV EXPERIMENTAL SETUP

FIGURE 5.

detector 1 was observed from detector 2. These results support the hypothesis of air Cerenkov. Since the air Cerenkov should be very broadband, no attempt was made at this time to measure the frequency of this radiation.

Based on these results, it was decided to enclose the dielectric slab in a vacuum chamber. Figure 6 is a photograph of the interior of the vacuum chamber. The vacuum was maintained in the chamber by installing a 1/16 in. thick piece of polyethelyne between two waveguide flanges exterior to the chamber. A waveguide run of about 75 ft. was added to the setup which allowed for the positioning of the detector in the LINAC control room thus eliminating the long, lossy coaxial cable runs. A Tektronic 491 Spectrum Analyzer (491-S/A) was obtained to facilitate frequency analysis.

The use of this new setup with team energy of 100.97 MeV, beam current of 4.3 nA and the same diode detector used before resulted in a 0.5 Volt peak pulse of about 1 microsecond duration. The increase in peak output was attributed to the reduction in line loss provided by the replacement of the coax cable by waveguide. Subsequent tests, which will be discussed later, have modified this hypothesis. The 491-S/A, with a 20 dB attenuator inserted between the waveguide to coax adapter and the S/A, detected signals at 12.39 GHz and 8.57 GHz. The 12.39 GHz signal was the image of a real signal above the frequency range of the 491-S/A. Using a relatively low Q cavity, it was determined that most of the



VACUUM CHAMBER

FIGURE 6.

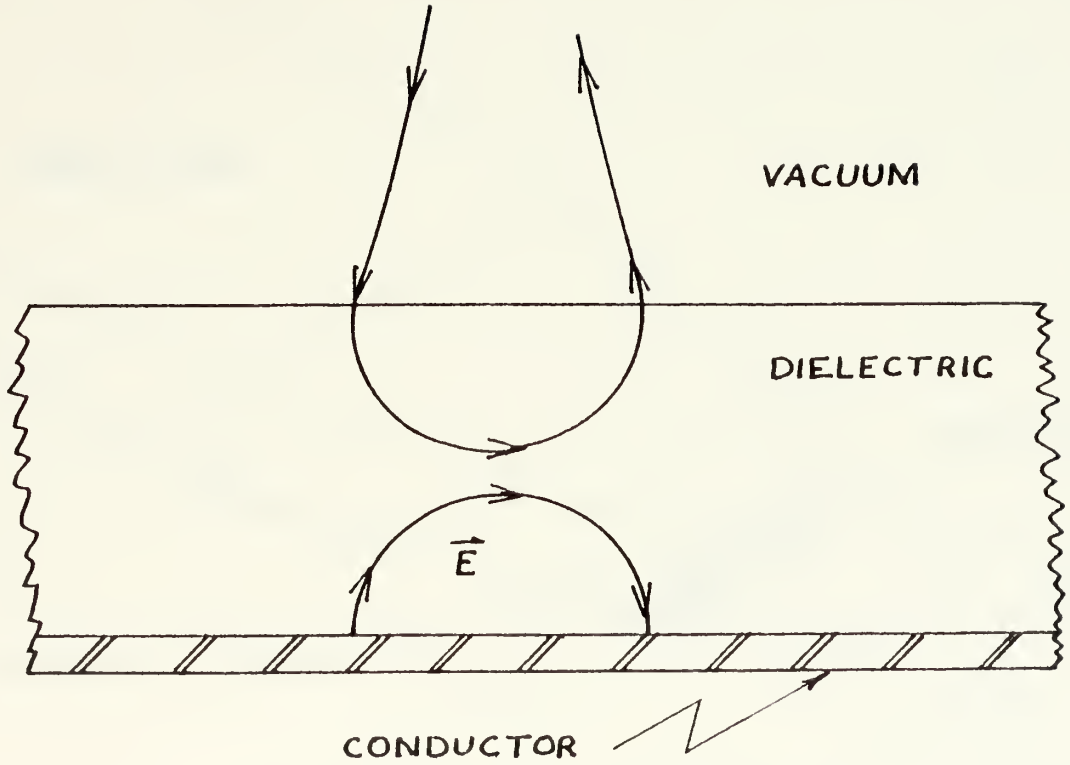
power in the pulse was in the 8.57 GHz signal. A dispersive high pass filter with a cutoff frequency of 9.9 GHz was installed between the waveguide from the dielectric and the coax adapter. With this arrangement and no attenuation at the input of the 491-S/A, valid signals were detected at 8.58, 8.99 and 11.42 GHz. When the dielectric was removed from the vacuum chamber and the test repeated, the same signals were detected. Both with and without dielectric, all signals disappeared when electron injection was interrupted with LINAC RF power maintained.

The 8.57 GHz and 11.48 GHz signals appear to be the third and fourth harmonics of the LINAC operating frequency. The exact cause of the 8.99 GHz signal is not known but it is believed to be caused by a resonant mode of the vacuum chamber. The chamber is a rectangular aluminum box with 10 mil aluminum windows for electron entry and exit and as such would act as a cavity resonator. Equipment limitations prevent the measurement of any change in strength of the 11.48 GHz signal with and without the dielectric slab in place. This frequency is only 4% above the predicted frequency for the given parameters and it is possible that the desired signal is hidden by the LINAC harmonic at that frequency. Also, the actual index of refraction of the polyethylene was not experimentally determined and it was not manufactured specifically for optical use. It is possible that a slight change in n and/or inhomogeneities in the material could cause some frequency shifting.

Additional tests were conducted with this configuration. Two anomalies were observed which required further explanation. First, an intermittent, valid signal was detected at 10.24 GHz. This signal is not fully understood but it is believed to be extraneous to the experiment. Secondly, the peak output of the diode detector varied as much as an order of magnitude with the same beam parameters and beam position.

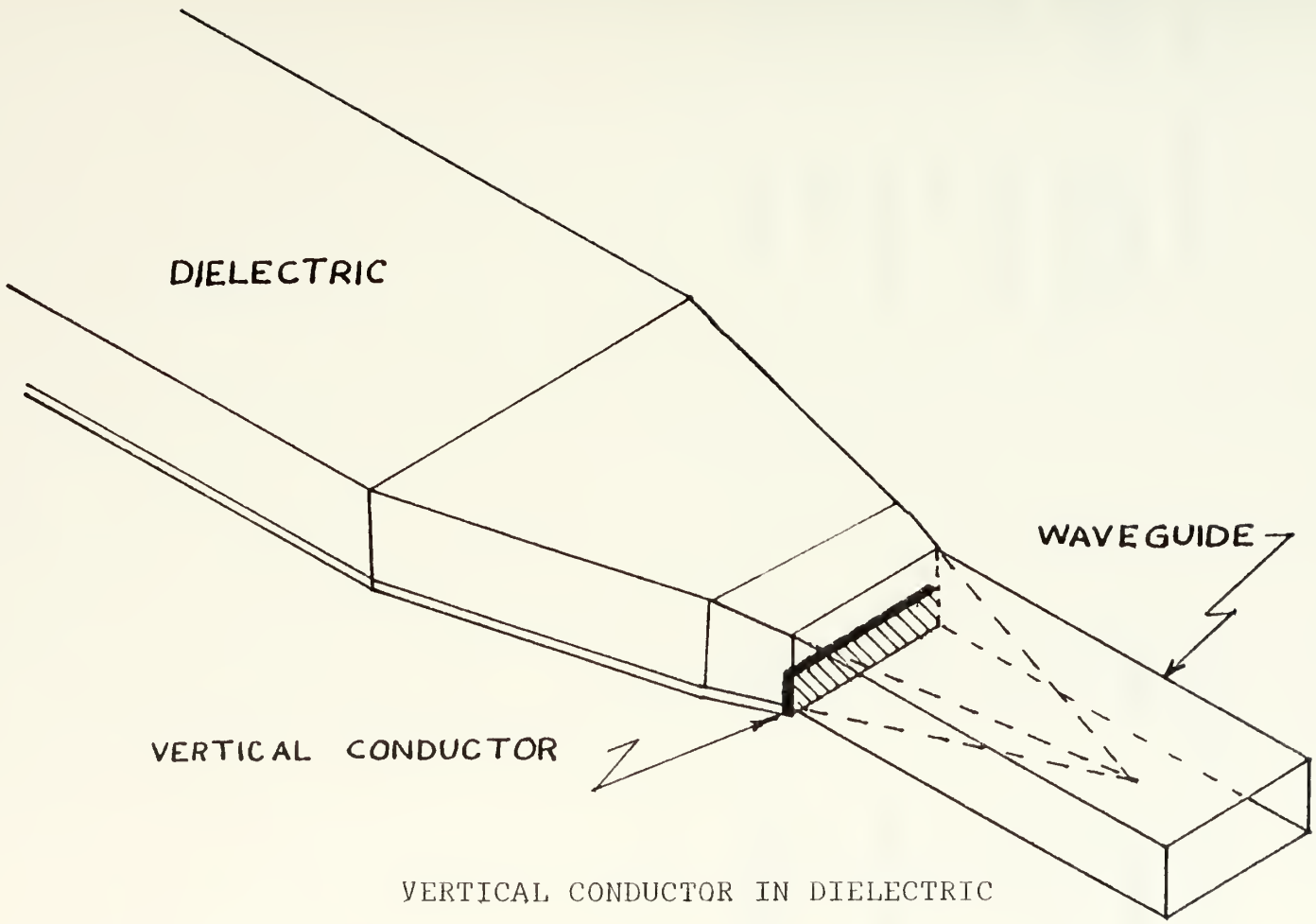
This observation led to a more intensive investigation of the TM to TE coupling at the dielectric/waveguide transition. In particular, for the m=1 mode, it was realized that cancellation of the E-field could occur at the transition point. A graphical representation of the E-field in the dielectric is shown in Figure 7. It was decided to nullify the lower field at the transition by inserting a vertical conductor into the dielectric slab at the mouth of the waveguide as shown in Figure 8. The vertical conductor should not affect any TE modes propagating in the dielectric but should nullify the cancellation effect of the m=1 mode for any TM mode waves propagating in the dielectric. Another modification of the setup was also made at this time. The 10 mil. aluminum window between the LINAC and the vacuum chamber was replaced by a 10 mil. mylar window to minimize beam spreading at the entrance to the vacuum chamber.

Table II shows the results of the frequency analysis of the dielectric with and without the conductor in place. The beam parameters were as follows: E=65.29 MeV, I=4.0 nA.




E-FIELD IN THE DIELECTRIC

FIGURE 7.




VERTICAL CONDUCTOR IN DIELECTRIC

FIGURE 8.

TABLE II

FREQUENCIES OBSERVED

| With Conductor | | Without Conductor | |
|---|---|---|---|
| f [GHz] | Strength | f [GHz] | Strength |
| 8.57 | very strong | 8.57 | strong |
| 9.00 | moderate | 9.00 | weak |
| 9.52 | weak | ---- | ---- |
| 10.21 | moderate | 10.25 | moderate |
| 10.92 | weak | ---- | ---- |
| 11.42 | strong | 11.50 | moderate |

The peak voltage observed from the diode detector was 150 mV without the vertical conductor and 100 mV with the conductor in place. It is interesting to note that although half of the area of the waveguide was blocked, the detector voltage dropped by only about one third.

For input values of E=65.29 MeV, n=1.461, h=12.7 mm and m=1, the TI-59 program predicts a frequency of 11.08 GHz. The 9.52 GHz signal does not correspond to any predicted frequency. Possible explanations of this frequency will be discussed later. The observed 10.92 GHz signal is only 1.4% below the predicted value. Since this frequency is only observed when the vertical conductor is in place, it is believed that this is a TM mode resulting from stimulated Cerenkov radiation.

## IV. DISCUSSION AND CONCLUSIONS

It appears from the foregoing experiment that stimulated Cerenkov radiation can be produced at the predicted frequency with the proper choice of parameters. The 1.4% error can be explained in several ways. First, the predicted frequency is based on an algorithm that provides an approximate solution for modes where m>0. Second, nonoptical quality polyethylene was used for the dielectric slab. Finally, the theoretical development assumed an infinite slab in the y direction. In fact, a finite slab was used whose y dimension was about 5 times the diameter of the focused beam. Any one of the deviations from the ideal case or a combination of them could account for the error. The most probable source of error is the index of refraction of the polyethylene. An index of refraction of 1.473, a change of 0.012 which is less than 1% of the tabulated value in Ref. 3, would reduce the error to essentially zero.

The cause of the TM mode frequency at 9.52 GHz shown in Table II is not fully understood. It could be a result of the edge effects caused by the finite width of the dielectric or it might be a resonant TM mode of the vacuum chamber.

A secondary effect observed during the course of this experiment is worth noting. When the focused beam was passed above the dielectric, the output of the diode detector

varied with the beam current. This effect was present both in air and in vacuum. It may be possible to construct a calibration curve of diode voltage vs. beam current and thereby develop a beam current measuring device which does not destroy the beam downstream from the measurement point as the beam current measuring device presently in use does.

It is readily apparent from the magnitude of the diode pulse that the stimulated Cerenkov radiation effect is very inefficient. To match the mode with high speed electrons requires that the component of the mode's E-field parallel to the electron beam be small compared with the field's transverse components. This will tend to make the coupling small. The output power of the diode detector is less than 0.5 mWatts. Most of this power, conservatively estimated at 95%, is in the 8.57 GHz signal as demonstrated in Section III. This does, however, enhance the possibility of developing an electron beam current meter, based on this process, which has only a minimal effect on the beam itself.

The results of this experiment support the hypothesis that stimulated Cerenkov radiation can be produced at a frequency specified by the physical parameters of electron energy, index of refraction and dielectric thickness. It appears that the wave-electron coupling is a weak effect which is further degraded by the TM to TE coupling used to extract the signal in this experiment. Greater efficiency may be possible if a more effective means of extracting the

signal is developed. The signal itself may be increased in strength by lengthening the dielectric slab to increase the length of the wave-electron interaction region.

Additional work in the area of stimulated Cerenkov radiation appears to be warranted. The development presented here is not guaranteed to be unique. There may be other modes that can be stimulated in the dielectric, one of which might provide a stronger signal. A mode with a larger longitudinal component could increase the electron-mode coupling. Additionally, the effect of the pulsed beam of electrons should be studied in depth. Specifically, the question of why this experimental setup produces most of its output power at the third harmonic of the LINAC operating frequency should be investigated in greater detail. If the power in the LINAC harmonics can be shifted to the Cerenkov frequency it would greatly increase the usefulness of the stimulated Cerenkov process as a radiation source. Finally, generation of X-band radiation using the m=0 mode should be attempted as a preliminary step in extending this work into the IR range. The dielectric thickness predicted by the theoretical development presented here for an IR generator is of the order of microns. The X-band m=0 thickness is in this range. Extracting a signal from a dielectric this thin may cause some difficulties. The experience thus gained should be valuable in extending this work into the IR range.

## APPENDIX A

## LINAC OPERATING CHARACTERISTICS

| | |
|---|---|
| Max Energy | 120 MeV |
| Max Average Current | 3-5 microAmp |
| Operating Frequency | 2.856 GHz |
| PRF | 60 pps |
| Pulse Duration | 1.0 microseconds |
| Nr. of Klystrons | 3 |
| Peak Output Power per Klystron | 21 MWatts |

## APPENDIX B

## TI-59 PROGRAM LISTING AND EXPLANATION

This program does not use any library programs within the algorithm. The inputs are the electron beam energy (in MeV), the index of refraction of the dielectric (dimensionless), either the desired frequency (in Hz) or the dielectric thickness (in meters), and the mode (dimensionless). The mode input determines which leg of the $y=(1/n^2)x \tan x$ in the curve (in the first quadrant) is used. For a given m=0,1,2... the solution is found for x values such that $m\pi<x<[(2m+1)\pi/2]$. As noted in Chapter II, if $x>\pi$, more than one solution is possible. This program solves only for the solution in the $m\pi<x<[(2m+1)\pi/2]$ range.

The solution for m=0 is unique and exact. Solutions for higher order modes are found using an iterative search. The solution is approximate in that the program searches down the designated leg of $y=(1/n^2)x \tan x$ in discrete steps until it finds a y value less than the y value for the corresponding m=0 case. This value of y is then taken as the solution point for the input mode.

To solve Equation 67:

1. Load program.
2. Enter E[MeV], press A.

3. Enter n, press B.

4. Enter f[nz], press C.

5. Enter m, press E.

The program will solve for h [meters].

To solve Equation 68:

1. Load program.

2. Enter E[MeV], press A.

3. Enter n, press B.

4. Enter h[m], press D.

5. Enter m, press E'.

The program will solve for f[Hz].

The program listing is on the following pages.

```
000  76 LBL
001  11 A
002  57 ENG
003  55 ÷
004  93 .
005  05 5
006  01 1
007  01 1
008  95 =
009  42 STO
010  00 00
011  91 R/S
012  76 LBL
013  12 B
014  33 x²
015  42 STO
016  01 01
017  65 X
018  08 8
019  93 .
020  08 8
021  05 5
022  04 4
023  01 1
024  01 1
025  08 8
026  52 EE
027  94 +/-
028  01 1
029  02 2
030  42 STO
031  02 02
032  95 =
033  42 STO
034  03 03
035  04 4
036  52 EE
037  94 +/-
038  07 7
039  65 X
040  89 π
041  95 =
042  42 STO
043  04 04
044  91 R/S
045  76 LBL
046  13 C
047  65 X
048  02 2
049  65 X
050  89 π
051  95 =
052  42 STO
053  05 05
054  91 R/S
055  76 LBL
056  14 D
057  42 STO
058  06 06
059  91 R/S
060  76 LBL
061  15 E
062  42 STO
063  07 07
064  67 EQ
065  19 D'
066  71 SBR
067  88 DMS
068  71 SBR
069  68 NOP
070  43 RCL
071  13 13
072  65 X
073  43 RCL
074  11 11
075  95 =
076  42 STO
077  14 14
078  71 SBR
079  58 FIX
080  42 RCL
081  15 15
082  33 x²
083  85 +
084  43 RCL
085  17 17
086  33 x²
087  95 =
088  55 ÷
089  53 (
090  43 RCL
091  10 10
092  75 -
093  43 RCL
094  09 09
095  54 )
096  95 =
097  34 √x
098  42 STO
099  18 18
100  91 R/S
101  76 LBL
102  18 C7
103  01 1
104  42 STO
105  05 05
106  71 SBR
107  88 DMS
108  71 SER
109  69 OP
110  55 ÷
111  02 2
112  55 ÷
113  89 π
114  95 =
115  91 R/S
116  76 LBL
117  19 D'
118  71 SBR
119  88 DMS
120  71 SBR
121  68 NOP
122  91 R/S
123  76 LBL
124  10 E'
125  42 STO
126  07 07
127  67 EQ
128  18 C'
129  01 1
130  42 STO
131  05 05
132  71 SBR
133  88 DMS
134  71 SBR
135  69 OP
136  43 RCL
137  08 08
138  75 -
139  43 RCL
140  09 09
141  95 =
142  34 √x
143  65 X
144  43 RCL
145  19 19
146  65 X
```

| | | |
|---|---|---|
| 147 | 43 | RCL |
| 148 | 06 | 06 |
| 149 | 95 | = |
| 150 | 42 | STO |
| 151 | 14 | 14 |
| 152 | 71 | SBR |
| 153 | 58 | FIX |
| 154 | 43 | RCL |
| 155 | 15 | 15 |
| 156 | 33 | $x^2$ |
| 157 | 85 | + |
| 158 | 43 | RCL |
| 159 | 17 | 17 |
| 160 | 33 | $x^2$ |
| 161 | 95 | = |
| 162 | 55 | ÷ |
| 163 | 43 | RCL |
| 164 | 06 | 06 |
| 165 | 33 | $x^2$ |
| 166 | 55 | ÷ |
| 167 | 53 | ( |
| 168 | 43 | RCL |
| 169 | 10 | 10 |
| 170 | 75 | - |
| 171 | 43 | RCL |
| 172 | 09 | 09 |
| 173 | 54 | ) |
| 174 | 95 | = |
| 175 | 34 | $\sqrt{x}$ |
| 176 | 42 | STO |
| 177 | 20 | 20 |
| 178 | 55 | ÷ |
| 179 | 02 | 2 |
| 180 | 55 | ÷ |
| 181 | 89 | π |
| 182 | 95 | = |
| 183 | 91 | R/S |
| 184 | 76 | LBL |
| 185 | 88 | DMS |
| 186 | 43 | RCL |
| 187 | 00 | 00 |
| 188 | 33 | $x^2$ |
| 189 | 35 | 1/x |
| 190 | 94 | +/- |
| 191 | 85 | + |
| 192 | 01 | 1 |
| 193 | 95 | = |
| 194 | 65 | X |
| 195 | 02 | 2 |
| 196 | 93 | . |
| 197 | 09 | 9 |
| 198 | 09 | 9 |
| 199 | 07 | 7 |
| 200 | 09 | 9 |
| 201 | 02 | 2 |
| 202 | 05 | 5 |
| 203 | 52 | EE |
| 204 | 08 | 8 |
| 205 | 33 | $x^2$ |
| 206 | 95 | = |
| 207 | 35 | 1/x |
| 208 | 65 | X |
| 209 | 43 | RCL |
| 210 | 05 | 05 |
| 211 | 33 | $x^2$ |
| 212 | 95 | - |
| 213 | 42 | STO |
| 214 | 08 | 08 |
| 215 | 43 | RCL |
| 216 | 05 | 05 |
| 217 | 33 | $x^2$ |
| 218 | 65 | X |
| 219 | 43 | RCL |
| 220 | 02 | 02 |
| 221 | 65 | X |
| 222 | 43 | RCL |
| 223 | 04 | 04 |
| 224 | 95 | = |
| 225 | 42 | STO |
| 226 | 09 | 09 |
| 227 | 43 | RCL |
| 228 | 05 | 05 |
| 229 | 33 | $x^2$ |
| 230 | 65 | X |
| 231 | 43 | RCL |
| 232 | 03 | 03 |
| 233 | 65 | X |
| 234 | 43 | RCL |
| 235 | 04 | 04 |
| 236 | 95 | = |
| 237 | 42 | STO |
| 238 | 10 | 10 |
| 239 | 43 | RCL |
| 240 | 08 | 08 |
| 241 | 75 | - |
| 242 | 43 | RCL |
| 243 | 09 | 09 |
| 244 | 95 | = |
| 245 | 34 | $\sqrt{x}$ |
| 246 | 42 | STO |
| 247 | 11 | 11 |
| 248 | 43 | RCL |
| 249 | 10 | 10 |
| 250 | 75 | - |
| 251 | 43 | RCL |
| 252 | 08 | 08 |
| 253 | 95 | = |
| 254 | 34 | $\sqrt{x}$ |
| 255 | 42 | STO |
| 256 | 12 | 12 |
| 257 | 92 | RTN |
| 258 | 76 | LBL |
| 259 | 68 | NOP |
| 260 | 43 | RCL |
| 261 | 01 | 01 |
| 262 | 65 | X |
| 263 | 43 | RCL |
| 264 | 11 | 11 |
| 265 | 55 | ÷ |
| 266 | 43 | RCL |
| 267 | 12 | 12 |
| 268 | 95 | = |
| 269 | 70 | RAD |
| 270 | 22 | INV |
| 271 | 30 | TAN |
| 272 | 55 | ÷ |
| 273 | 43 | RCL |
| 274 | 12 | 12 |
| 275 | 95 | = |
| 276 | 42 | STO |
| 277 | 13 | 13 |
| 278 | 92 | RTN |
| 279 | 76 | LBL |
| 280 | 69 | OP |
| 281 | 43 | RCL |
| 282 | 01 | 01 |
| 283 | 65 | X |
| 284 | 43 | RCL |
| 285 | 11 | 11 |
| 286 | 55 | ÷ |
| 287 | 43 | RCL |
| 288 | 12 | 12 |
| 289 | 95 | = |
| 290 | 70 | RAD |
| 291 | 22 | INV |
| 292 | 30 | TAN |
| 293 | 55 | ÷ |

```
294  53 (
295  43 RCL
296  12 12
297  65 X
298  43 RCL
299  06 06
300  54 )
301  95 =
302  42 STO
303  19 19
304  92 RTN
205  76 LBL
306  58 FIX
207  09 9
308  94 +/-
309  22 INV
310  28 LOG
311  85 +
312  43 RCL
313  07 07
314  65 X
315  89 π
316  95 =
317  42 STO
318  15 15
319  01 1
320  01 1
321  94 +/-
322  22 INV
232  28 LOG
324  94 +/-
325  42 STO
326  16 16
327  76 LBL
328  50 |x|
329  43 RCL
330  15 15
331  70 RAD
332  30 TAN
333  65 X
334  43 RCL
335  15 15
336  55 ÷
337  43 RCL
338  01 01
339  95 =
340  42 STO
341  17 17
342  75 -
343  43 RCL
344  14 14
345  95 =
346  22 INV
347  77 GE
348  59 INT
349  43 RCL
350  16 16
351  44 SUM
352  15 15
353  61 GTO
354  50 |x|
355  76 LBL
356  59 INT
357  92 RTN
```

## LIST OF REFERENCES


1. Walsh, J. E., "Stimulated Cerenkov Radiation," Physics of Quantum Electronics, Vol. 5, Ed. S. F. Jacobs et al, Addison Wesley Publishing Company, Inc., 1978.

2. Simonis, G. J., "Near mm Wave Materials," Harry Diamond Labs.

3. Hodgman, C. D., Handbook of Chemistry and Physics, 38th Edition, Chemical Rubber Publishing Co., 1956.

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